Amplification of Hypersound in Graphene with degenerate energy dispersion

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Abstract

Hypersound amplification/absorption of acoustic phonons in Graphene with degenerate energy dispersion $\varepsilon(p)$ near the Fermi level was theoretically studied. For $k_\beta T << 1$ and $ql >> 1$, the dependence of the absorption coefficient Γ/Γ_0 on $\frac{V_D}{V_s}$ was studied where the results satisfied the Cerenkov effect. That is when $\frac{V_D}{V_s} > 1$, an amplification was obtained but for $\frac{V_D}{V_s} < 1$, an absorption was obtained which could lead to Acoustoelectric Effect (AE) in Graphene. A linear dependence of the Γ/Γ_0 on ω_q was observed where the result obtianed qualitatively agreed with an experimentally observed acoustoelectric current in Graphene via the Weinrich relation. It is interesting to note from this study that, frequencies above $10THz$ can be attained for $V_D = 1.1ms^{-1}$. This study permit the use of Graphene as hypersound phonon laser (SASER).

Introduction

Graphene, a member of the Carbon allotropes has exceptional properties for future nanoelectronics [1, 2, 3, 4]. It is an ideal two-dimensional electron

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gas ($2DEG$) system made up of one layer of Carbon atom having a high electron mobility (μ) at room temperature with high mechanical and thermodynamic stability [5]. Several unusual phenomena such as half-integer quantum Hall effect [6], non-zero Berry's phase [7], and minimum conductivity [8] have been observed experimentally in Graphene. The most interesting property of Graphene is its linear energy dispersion $E = \pm\hbar V_F|k|$ (the Fermi velocity $V_F \approx 10^8 ms^{-1}$) at the Fermi level with low-energy excitation. This makes graphenes applicable in advance electronics and optoelectronic devices such as sub-terahertz Field-effect transistors [9], infrared transparent electrodes [10] and THz plasmonic deives [11]. Currently, among the various studies on Graphene attracting much attention is the generation and detection of hypersound amplification or absorption of acoustic phonons [12]. It is known that, when an acoustic phonon passes through a semiconductor, it may interact with various elemental excitations which may lead to amplifcation or absorption of the phonons. The idea of acoustic wave amplification in bulk material was theoretically predicted by Tolpygo (1956), Uritskii [13], and Weinreich [14] and in N-Ge by Pomerantz [15]. Hypersound generation in bulk [16] and low-dimensional materials such as Superlattices [17, 18, 19, 20], Cylindrical Quantum Wire [21], Quantum Wells [22] and Graphenes Nanoribbons (GNR) [23] have been studied. Akin to Cerenkov acoustic-phonon emission, when the drift velocity of electrons V_D exceeds the sound velocity (V_s) of the host material [24] lead to amplification of the acoustic-phonons or when $V_D < V_s$ causes absorption. This has been ultilised experimentally to confirm the breakdown of quantum Hall effect [25], the generation of coherent phonon-polariton radiation [26], and large acoustic gain in coherent phonon

oscillators in semiconductors [27]. Furthermore, the emission and absorption of acoustic-phonons is used to provide detailed information on the excitation and relaxation mechanisms in semiconductors via deformation potential, where the effect of interactions can be used to determine the physical properties of the material. In particular, acoustic-phonons providing terahertz ($10^{12} Hz$) hypersonic sources can lead to the attainment of phonon laser or SASER [28, 29] in graphene via Cerenkov effect which is an intense field of research. Following the works of Nunes and Fonseca [32], Zhao et. al [33] proposed the possibility of attaining Cerenkov acoustic-phonon emission in Graphene whilst Insepov et. al [31], performed experimentally the surface acoustic wave Amplification by D.C voltage supply in Graphene. In this paper, unlike [32], where when $V_D = 0$, expression goes to zero, in otherwords there is no absorption. Here, we have a general expresion which acounts for absorption when $V_D = 0$ or $V_D < V_s$. This leads to the observation of acoustoelectric effect. This has been verified experimentally in Graphene [37]. The motivation for this work is to provide the theoretical framework that can lead to the attainment of SASER in Graphene, for use as a phonon spectrometer, for generation of high-frequency electric oscillation, and as a non-destructive testing of microstructure and acoustic scanning system. The paper is organised as follows: In theory section, the theory underlying the amplification (Absorption) of acoustic-phonon via Cerenkov effect is presented. In the numerical analysis section, the final equation is analysed and presented in a graphical form. Lastly, the conclusion is presented in section 4.

Theory

We will proceed following the works of [32], here the acoustic wave will be consisdered as phonons of frequency (ω_q) in the short-wave region $ql >> 1$ (q is the acoustic wave number, l is the electron mean free path). The kinetic equation for the acoustic phonon population $N_{\vec{q}}(t)$ in the graphene sheet is given by

$$\frac{\partial N_{\vec{q}}}{\partial t} = \frac{2\pi}{\hbar} g_s g_v \sum_{k,k'} |C_{\vec{q}}|^2 \delta_{k,k'+\vec{q}} \{[N_{\vec{q}}(t)+1] f_{\vec{k}}(1-f_{\vec{k}'})\delta(\varepsilon_{\vec{k}'} - \varepsilon_{\vec{k}} + \hbar\omega_{\vec{q}}) \\ - N_{\vec{q}}(t) f_{\vec{k}'}(1-f_{\vec{k}})\delta(\varepsilon_{\vec{k}'} - \varepsilon_{\vec{k}} - \hbar\omega_{\vec{q}})\} \quad (1)$$

where $g_s = g_v = 2$ account the for spin and valley degeneracies respectively, $N_{\vec{q}}(t)$ represent the number of phonons with a wave vector $\vec{q}$ at time t. The factor $N_{\vec{q}} + 1$ accounts for the presence of $N_{\vec{q}}$ phonons in the system when the additional phonon is emitted. The $f_{\vec{k}}(1 - f_{\vec{k}})$ represent the probability that the initial $\vec{k}$ state is occupied and the final electron state $\vec{k}'$ is empty whilst the factor $N_{\vec{q}} f_{\vec{k}'}(1 - f_{\vec{k}})$ is that of the boson and fermion statistics. The unperturbed electron distribution function is given by the shifted Fermi-Dirac function as

$$f_{\vec{p}} = [exp(-\beta(\varepsilon(\vec{p} - mv_D) - \chi))]^{-1} \quad (2)$$

where $f_{\vec{p}}$ is the Fermi-Dirac equilibrium function, with χ being the chemical potential, $\vec{p}$ is momentum of the electron, $\beta = 1/kT$, k is the Boltzmann constant and V_D is the net drift velocity relative to the ion lattice site. In Eqn (1), the summation over k and k' can be transformed into integrals by

the prescription

$$\sum_{k,k'} \rightarrow \frac{A^2}{(2\pi)^4} \int d^2k d^2k'$$

where A is the area of the sample, and assuming that $N_q(t) >> 1$ yields

$$\frac{\partial N_{\vec{q}}}{\partial t} = \Gamma_{\vec{q}} N_{\vec{q}} \tag{3}$$

where

$$\Gamma_{\vec{q}} = \frac{A|\Lambda|^2 \hbar q}{(2\pi)^3 \hbar V_F \rho V_s} \int_0^\infty kdk \int_0^\infty k'dk' \int_0^{2\pi} d\phi \int_0^{2\pi} d\theta \{[f(k) - f(k')] \\ \delta(k - k' - \frac{1}{\hbar V_F}(\hbar\omega_q - V_D \cdot \hbar\vec{q}))\} \tag{4}$$

with $k' = k - \frac{1}{\hbar V_F}(\hbar\omega_q - V_D \cdot \hbar\vec{q})$. Λ is the deformation potential constant, and ρ is the density of the graphene sheet. At low temperature $k_B T << 1$, the distribution function become $f(k) = exp(-\beta(\varepsilon(k)))$. Eqn(4) can be expressed as

$$\Gamma_{\vec{q}} = \frac{A|\Lambda|^2 \hbar q}{(2\pi) \hbar V_F \rho V_s} \int_0^\infty kdk(k - \frac{1}{\hbar V_F}(\hbar\omega_q - V_D \cdot \hbar\vec{q}))[exp(-\beta\hbar V_F k) \\ -exp(-\beta\hbar V_F(k - \frac{1}{\hbar V_F}(\hbar\omega_q - V_D \cdot \hbar\vec{q}))] \tag{5}$$

Using standard intergrals, Eqn(5) can be expressed finally

as

$$\Gamma = \Gamma_0\{2 - \beta\hbar\omega_q(1 - \frac{V_D}{V_s})\}[1 - exp(-\beta\hbar\omega_q(1 - \frac{V_D}{V_s}))] \tag{6}$$

where

$$\Gamma_0 = \frac{A|\Lambda|^2 q}{(2\pi)\beta^3 \hbar^3 {V_F}^4 \rho V_s} \tag{7}$$

Numerical Analysis

The Eqn (6) is analysed numerically for a normalized graph of $\frac{\Gamma}{\Gamma_0}$ against $\frac{V_D}{V_s}$ and ω_q. The following parameters were used $\Lambda = 9eV$, $T = 10K$, $V_s = 2.1 \times 10^6 cms^{-1}$ and $\vec{q} = 10^5 cm^{-1}$. In Figure 1, the graph for the dependence of $\frac{\Gamma}{\Gamma_0}$ on ω_q is plotted. The graph was obtained at $\frac{V_D}{V_s} < 1$. The insert shows an experimentally obtained graph of an acoustoelectric current for gate-controlled Graphene [37]. The hypersound absorption graph qualitively agreed with the experimentally obtained graph via the Weinriech relation [36]. In Figure $2a$, the dependence of $\frac{\Gamma}{\Gamma_0}$ on $\frac{V_D}{V_s}$ is analysed. From the graph, when $\frac{V_D}{V_s} > 1$, gives rise to an amplification as also indicated in the work of Nunes and Fonseca [32], however, when $\frac{V_D}{V_s} < 1$, an absorption of acoustic phonons was observed. To enhanced the observed Amplification (Absorption), a $3D$ graph was plotted for frequencies $\omega_q = 0.2$, 0.4, and $1THz$ (see Figure $2b$, 3 and 4). In Figure $2b$, the maximum amplification was obtained at $\frac{\Gamma}{\Gamma_0} = -0.16$ at $\omega_q = 2THz$ for $V_D = 1.1V_s$. For figure $3(a)$, at $V_D = 1.1V_s$, $\frac{\Gamma}{\Gamma_0} = -0.34$ whislt in figure $3(b)$, for $V_D = 1.1V_s$, $\frac{\Gamma}{\Gamma_0} = -0.08$ was obtained. It is interesting to note that, acoustic-phonon frequencies above $10THz$ can be attained. In Figure 4, at $V_D = 1.1V_s$, gave $\frac{\Gamma}{\Gamma_0} = -3.17$ which was obtained at $\omega_q = 20THz$

For graphene, with $V_D = 1.1V_s$, the field E can be calculated since $E = V_D/\mu$. The electron mobility μ in graphene given as $2.0 \times 10^4 cm^2/Vs$, $V_s = 2.1 \times 10^5 cm/s$ gives $E = 11.5V/cm$.

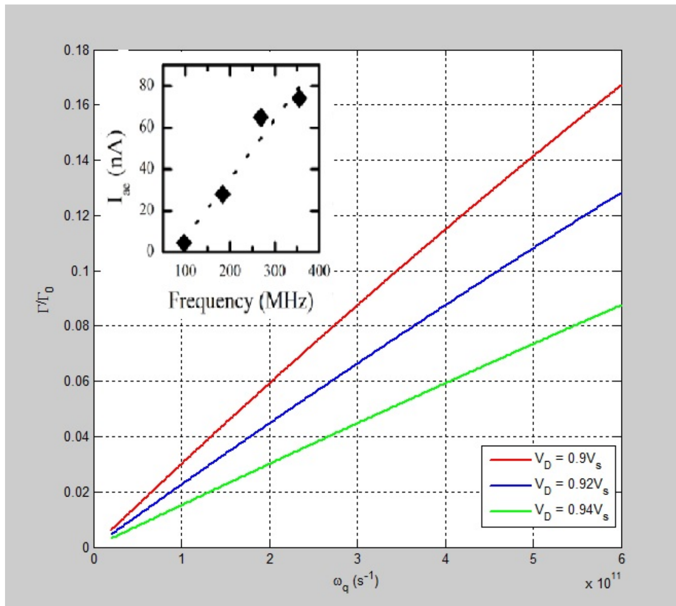

Figure 1: Dependence of Γ/Γ_0 on ω_q insert is the experimental verification of Acousto-electric current versus acoustic phonon frequency [37]

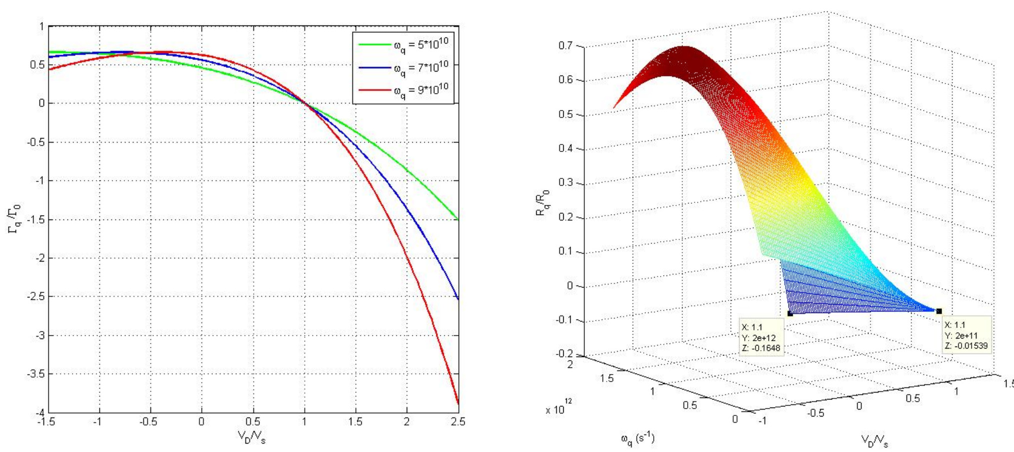

Figure 2: (a) Dependence of Γ/Γ_0 on $\frac{V_D}{V_s}$ for varying ω_q (left) (b) $3D$ representation of Γ/Γ_0 on $\frac{V_D}{V_s}$ and ω_q at $0.2THz$ (right)

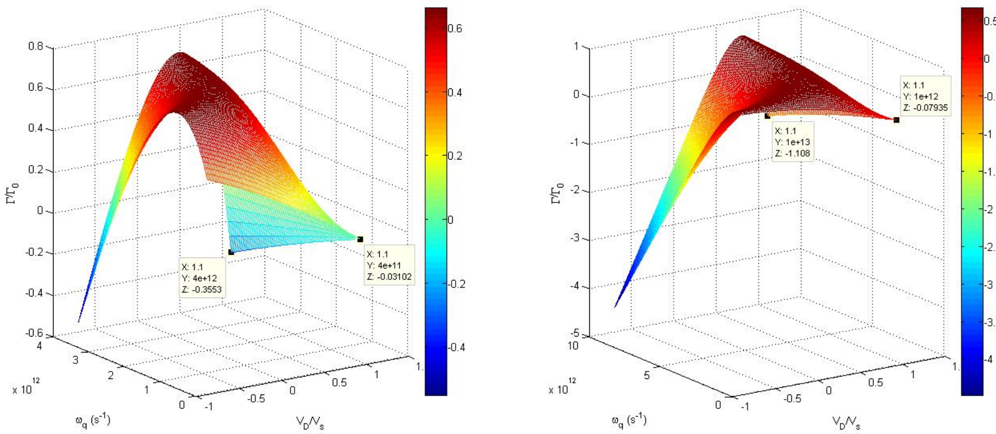

Figure 3: $3D$ representation of Γ/Γ_0 on $\frac{V_D}{V_s}$ and ω_q at (a) $0.4THz$ (left) and at $1THz$(right)

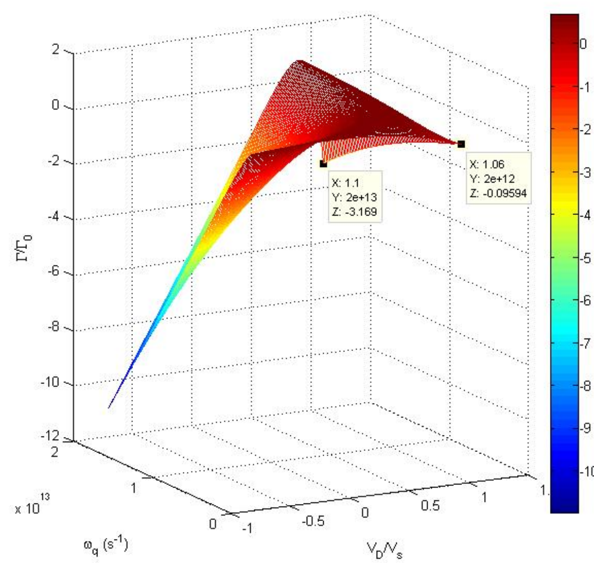

Figure 4: A graph of Γ/Γ_0 on $\frac{V_D}{V_s}$ and ω_q at $2THz$

Conclusion

The generation of hypersound amplification (absorption) of acoustic - phonons in Graphene is studied. For $\frac{V_D}{V_s} > 1$, the hypersound amplification obtained is similar to that of Nunes and Fonseca but for $\frac{V_D}{V_s} < 1$, an absorption is obtained which could lead to Acoustoelectric Effect in Graphene. The absorption obtained qualitatively agreed with an experimentally obtained acoustoelectric current in a graphene via the Weinrich relation. For a drift velocity of $V_D = 1.1V_s$, a field of $E = 11.5V/cm$ was calculated. At frequency of $0.2THz$, an amplification of $\Gamma/\Gamma_0 = -3.17$ is attained. From this work, the hypersound studies in graphene offers a much better source of higher phonon frequencies than the homogenous semiconductors which permit the use of graphene as hypersound phonon laser (SASER).

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